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ON THE GENERALITY OF SWITCHING NETWORKS

WITH RESTRICTED NON-LINEARITY

A. C. McKellar

Summary

The generality of special types of asynchronous realizations of sequential machines is considered and compared with the synchronous case. It is shown that linear asynchronous networks are essentially combinational. On the other hand, every sequential machine can be realized as a network whose output and feedback functions are linear in the feedback inputs. It is proven that more feedback loops are required to do this than are required by known synchronous realizations. A relationship is established between asynchronous networks which are quadratic in their feedback inputs and synchronous networks which are linear in their feedback inputs.

Introduction

The concept of a linear switching network, which was introduced by Huffman [1], has been studied extensively. Unfortunately, the set of sequential machines which can be realized by linear switching networks is rather small, as we shall show. Seshu [2] proposed an extension to quasi-linear switching networks, i.e. networks which are linear in their feedback variables. Chang [3] extended many of the results concerning linear switching networks to the quasi-linear case and showed that, provided one was willing to use a sufficient number of feedback loops, any flow table could be realized as a synchronous, quasi-linear switching network.

In this paper, the generality of such realizations is considered for the asynchronous case. First, it is shown that a non-trivial sequential machine cannot be realized as a linear switching network. On the other hand, every flow table can be realized as a quasi-linear switching network. This is proven by showing that a combination of Huffman's one relay per row realization [4] and a version of the Hamming row set assignment procedure [5] yields a quasi-linear realization. Further, it is shown that there are flow tables which do not have an asynchronous, quasi-linear realization using as few feedback loops as Chang's canonical, synchronous, quasi-linear realization [3].

Preparata [6] has obtained an interesting description of state-logic relations which utilizes matrix algebra. He uses a rather large set of basis functions and this suggested inquiry into what could be done with a smaller set. Accordingly, the notion of a quasi-quadratic switching network is defined and is shown to be general for the asynchronous case by a procedure closely related to Chang's canonical quasi-linear procedure.

The model and notation will be consistent with Huffman [1]. Since we are concerned with asynchronous realizations, it is assumed that the specification contains no cycles, i.e. for a given primary input and internal state, the machine must go to a unique stable state.

Linear Switching Networks

A linear switching network is one for which the primary and feedback outputs are linear functions of the primary and feedback inputs. Provided such a network has no critical races [4], we can prove

Lemma 1: If a change in some input line to a linear switching network causes a subset of the outputs to change, then a change in that input line always causes exactly that subset of the outputs to change.

Proof: The outputs are expressible as

$$Y_i = a_{i0} \oplus \sum_{j=1}^{m} a_{ij} x_j \oplus \sum_{j=1}^{n} b_{ij} y_j$$

$$z_i = c_{i0} \oplus \sum_{j=1}^{m} c_{ij} x_j \oplus \sum_{j=1}^{n} d_{ij} y_j$$

where the a's, b's, c's, and d's are constant with values of 0 or 1 and $\oplus$ and Σ are "exclusive or" or addition modulo 2. A change in an input will cause an output to change if and only if the corresponding coefficient is 1. Since the final state of the network is required to be independent of the order in which the feedback inputs change whenever more than one is trying to change simultaneously, it follows that the set of outputs which change in reaching the final state is uniquely determined by the input which changes.

We will say that a sequential machine is trivial if the present output is a single valued function of the present input and the initial state.

Theorem 1: Any sequential machine which has a linear asynchronous realization is trivial.

Proof: Consider any input sequence, $(I_1, \ldots, I_n = I_1)$ applied to a linear asynchronous switching network. Since the last input is the same as the first, every input line changes an even number of times. Thus the realization must return to its original state and therefore the sequential machine which is realized is trivial.

Quasi-Linear Switching Networks

A switching network is defined to be quasi-linear if the feedback and primary outputs can be written in the form

$$Y_i = \sum_{j=1}^{n} F_{ij} y_j \oplus u_i$$

$$z_i = \sum_{j=1}^{n} H_{ij} y_j \oplus v_i$$

or in matrix notation

$$\begin{aligned} \underset{\sim}{Y} &= \underset{\sim}{F}\,\underset{\sim}{y} \oplus \underset{\sim}{u} \\ \underset{\sim}{z} &= \underset{\sim}{H}\,\underset{\sim}{y} \oplus \underset{\sim}{v} \end{aligned} \tag{1}$$

where $\oplus$ and Σ are "exclusive or" and the entries of $\underset{\sim}{F}$, $\underset{\sim}{H}$, $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are Boolean functions of the primary inputs, $x_1, \ldots, x_m$.

The sequential machine to be realized is assumed to be given as a flow table, T, (completely specified, otherwise we can make it so) with q rows which are numbered from 0 to q-1. We embed T in a flow table, S, which has 2^q rows numbered from 0 to 2^q-1 by placing row i of T in row 2^i of S. Thus S has 2^q-q don't care rows.

We now employ the Hamming row set assignment procedure [5] to code the rows of S. Consider the 2^q-1 dimensional vector space, V, over the field modulo 2. With each vector $\alpha = (a_1, \ldots, a_{2^q-1})$ in V, we associate a binary number of length q in the following way. The i^{th} significant bit of the number is the sum modulo 2 of those components of α which have a 1 in the i^{th} significant bit of their position number. The set of vectors in V which have the number j associated with them are assigned to row j of S.

The transitions will now be defined for each primary input, I_i, by means of a homomorphism, F_i, on V into V. Specifically, for each state α and each input, I_i, the next state is $F_i(\alpha)$. Thus quasi-linearity will be automatic and we will have to show only that F_i satisfies the transitions without critical races.

Consider a particular column of S, i.e. a fixed primary input, I_i. Let $b_1, \ldots, b_\ell$ be the numbers of the rows in S which contain an unstable state and $c_1, \ldots, c_\ell$ be the corresponding numbers of the row(s) to which they go under I_i. Clearly, each b_k and c_k is a power of 2. Define d_k by

$$d_k = b_k + c_k \qquad\qquad k = 1, \ldots, \ell$$

where + is addition in the integers. The homomorphism, F_i, is defined by

$$F_i(y_j) = \begin{cases} 0 & j \in \{d_k\} \\ y_j \oplus \sum_{k=1}^{\ell} e_{j,d_k} y_{d_k} & \text{otherwise} \end{cases} \tag{2}$$

where

$$e_{j,d_k} = \begin{cases} 1 & \text{if the } \log_2(b_k) \text{ bit of } j \text{ is } 1 \\ 0 & \text{otherwise} \end{cases}$$

and y_j designates the elementary vector which has a one in position j as well as a feedback input since no confusion will result.

The binary representation of d_k has a 1 in exactly two positions, one of which corresponds to b_k and the other to c_k. Since row b_k of S contains an unstable state for I_i whereas row c_k contains a stable state, d_k determines b_k uniquely. Therefore, F_i is well defined.

Let $\alpha = (a_1, \ldots, a_{2^q-1})$ by any vector in V. Then

$$F_i(\alpha) = F_i\left(\sum_{j=1}^{2^q-1} a_j y_j\right)$$

$$= \sum_{j=1}^{2^q-1} a_j F_i(y_j) \quad . \tag{3}$$

Substituting Equation (2) into Equation (3) gives

$$F_i(\alpha) = \sum_{j \notin \{d_k\}} a_j y_j \oplus \sum_{j \notin \{d_k\}} a_j \sum_{k=1}^{\ell} e_{j,d_k} y_{d_k}$$

$$= \sum_{j \notin \{d_k\}} a_j y_j \oplus \sum_{k=1}^{\ell} \left[\sum_{j \notin \{d_k\}} a_j e_{j,d_k} \right] y_{d_k} \quad .$$

If $e_{j,d_k} = 1$, then either $j = d_k$ or $j \notin \{d_k\}$. Thus

$$F_i(\alpha) = \sum_{j \notin \{d_k\}} a_j y_j \oplus \sum_{k=1}^{\ell} \Big[\sum_{j \neq d_k} a_j e_{j,d_k} \Big] y_{d_k} \qquad (4)$$

Clearly, F_i has no effect on α with the possible exception of components d_k.

The following lemma shows that F_i satisfies the transitions without critical races.

<u>Lemma 2</u>: Let α be a vector assigned to a row of S which is also a row of T and let I_i be a fixed primary input. Then, $F_i(\alpha)$ is the next state and $F_i(\alpha)$ and α differ in at most one component.

<u>Proof</u>: We consider two cases.

Case 1: α is assigned to row 2^m which is stable under I_i.

There must be a 1 in an even number of components which have a 1 in bit $\log_2(b_k)$ of their position number since $2^m \notin \{b_k\}$. Thus, if $a_{d_k} = 0$, then there are an even number of j different from d_k such that $a_j e_{j,d_k} = 1$ and hence

$$\sum_{j \neq d_k} a_j e_{j,d_k} = 0 = a_{d_k} \quad .$$

If $a_{d_k} = 1$, then there are an odd number of j different from d_k such that $a_j e_{j,d_k} = 1$ and hence

$$\sum_{j \neq d_k} a_j e_{j,d_k} = 1 = a_{d_k} \quad .$$

Therefore, upon substituting into Equation (4), we have $F_i(\alpha) = \alpha$, as required.

Case 2: α is assigned to row b_m which undergoes a transition to row c_m under I_i.

The proof that F_i leaves component d_k, $k \neq m$, of α invariant is essentially the same as for Case 1.

There must be a 1 in an odd number of components which have a 1 in bit $\log_2(b_m)$ of their position number. With appropriate modifications to the argument for Case 1, we have

$$\sum_{j \neq d_m} a_j e_{j,d_m} = 1 \oplus a_{d_m} .$$

Therefore $F_i(\alpha) = \alpha \oplus y_{d_m}$. It is easy to see that $F_i(\alpha)$ is assigned to row c_m of S and hence $F_i(F_i(\alpha)) = F_i(\alpha)$ by Case 1. $F_i(\alpha)$ and α differ in exactly one component. This completes the proof.

We have essentially proven

Theorem 2: Every sequential machine can be realized as an quasi-linear asynchronous switching network.

The matrix representation of the homomorphism, F_i, relative to the standard ordered basis for V is equal to the matrix F in Equation (1) evaluated at I_i. The realization is given by

$$\underset{\sim}{F} = \sum_{\text{all } i} g_i \underset{\sim}{F}_i$$

$$\underset{\sim}{u} = \underset{\sim}{0}$$

where g_i is the Boolean function of the primary inputs whose value is 1 if $I = I_i$ and 0 otherwise. It is not difficult to obtain a representation of the required form for the output functions, z_i.

This realization in its normal mode of operation (the portion corresponding to flow table T) is totally sequential and the transitions are satisfied in exactly the same way as the Hamming row set assignment procedure. It can be shown that the overall machine has no critical races.

We could have done essentially the same thing if we had embedded T in a flow table, S, with 2^{q-1} rows where row zero of T would appear as row zero of S. This was not done because it complicates the proof. This change would cure an objectionable property of the realization given here, viz, if the realization starts in the zero state, it can never get to any other state.

A question arises as to what can be done if we place further restrictions on the realization. In view of Chang's q-1 feedback loop quasi-linear realization of a q row flow table, it is interesting to ask whether or not we can do the same thing asynchronously. The following argument can be formalized to prove

<u>Theorem 3</u>: There exists a q row flow table, $q \geq 3$, for which there is no asynchronous quasi-linear realization using at most q-1 feedback loops.

It is not difficult to construct a 3 row flow table and show exhaustively that there is no quasi-linear realization with 2 feedback loops. Suppose there exists a q row flow table, T, which cannot be realized with q-1 feedback loops. We now construct a q+1 row flow table which cannot be realized with q feedback loops.

There are a finite number, N, of quasi-linear realizations (a realization is completely specified) of T which have q feedback loops. If N = 0 we are done by embedding T in any q+1 row flow table. Otherwise we take $2^q N$ copies of T side by side. To this new flow table we add a row in the following way. The portion of the new row appearing in the columns occupied by the i^{th} copy of T must be inconsistent with the i^{th} state of the first realization of T, $1 \leq i \leq 2^q$. We do exactly the same thing for each of the N possible realizations of T.

It is not difficult to arrange the primary inputs such that by fixing the value of some subset, we can restrict to any copy of T we choose. The existence of a quasi-linear realization with q feedback loops for this q+1 row table leads to an immediate contradiction.

Quasi-Quadratic Switching Networks

We will say that a sequential machine is quasi-quadratic if there exists a realization whose feedback and primary outputs can be written in the form

$$Y_i = u_i \oplus \sum_{1 \leq j \leq k \leq n} f_{ijk}\, y_j y_k$$

$$z_i = v_i \oplus \sum_{1 \leq j \leq k \leq n} h_{ijk}\, y_j y_k$$

where the f's, h's, u's, and v's are again Boolean functions of the primary input.

It is easy to see that we are free to choose the value of each Y_i and z_i for every state in which at most two of the feedback inputs, y_j, are non-zero. The value of the Y_i and z_i for any other state is a linear combination of the chosen values. The following synthesis procedure uses only those states in which we are free to choose the Y_i and z_i and therefore can be made quasi-quadratic by a suitable choice of don't care conditions.

Let a flow table with q+1 rows be given. Code the rows with the zero vector and the elementary vectors (elements of the standard basis) from the q dimensional vector space over the field modulo 2.

A transition between the zero row and any other row can be made to go directly by making exactly one feedback line change. A transition from a row in which $y_i = 1$ to a row in which $y_j = 1$ will be made to go by way of the state in which $y_i = y_j = 1$. This is totally sequential since y_j changes from 0 to 1 and then y_i changes from 1 to 0. Therefore by suitable choosing the remaining don't care conditions, we have an asynchronous, quasi-quadratic realization.

Thus we have proven

Theorem 4: Every q row flow table can be realized by a quasi-quadratic switching network which uses at most q-1 feedback loops.

The coding of the rows of the flow table is exactly the same as for Chang's canonical realization [3]. The realization is a very minor modification of Huffman's one-relay-per-row realization [1] which is also quasi-quadratic.

Concluding Remarks

The argument which was used to prove that a non-trivial sequential machine cannot be realized as a linear asynchronous switching network can be extended to include networks which are speed independent [7]. That the situation is different for synchronous linear networks is due to the fact that a wider class of flow tables can be admitted. It is rather surprising that restricting to quasi-linear realizations does not define a proper subclass of sequential machines. Clearly, every autonomous sequential machine can be realized as a linear switching network.

It is interesting to note that by restricting the allowable realizations even further, viz. at most q-1 feedback loops for a q row flow table, we obtain a distinction between synchronous and asynchronous networks. An open question in this regard is: What happens if we restrict to realizations which have only one state per row?

The quasi-quadratic realization described here bears a striking resemblance to Chang's canonical quasi-linear realization. In this sense, quasi-quadratic switching networks seem to be the natural asynchronous counterpart of synchronous quasi-linear switching networks.

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The generality of special types of asynchronous realizations of sequential machines is considered and compared with the synchronous case. It is shown that linear asynchronous networks are essentially combinational. On the other hand, every sequential machine can be realized as a network whose output and feedback functions are linear in the feedback inputs. It is proven that more feedback loops are required to do this than are required by known synchronous realizations. A relationship is established between asynchronous networks which are quadratic in their feedback inputs and synchronous networks which are linear in their feedback inputs.

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